3.3 Prove Lines are Parallel

Before You used properties of parallel lines to determine angle relationships.
Now You will use angle relationships to prove that lines are parallel.
Why? So you can describe how sports equipment is arranged, as in Ex. 32.

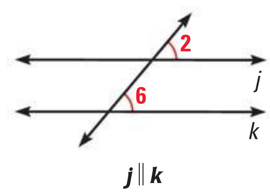
Key Vocabulary
- **paragraph proof**
- **converse,** *p. 80*
- **two-column proof,** *p. 112*

Postulate 16 below is the converse of Postulate 15 in Lesson 3.2. Similarly, the theorems in Lesson 3.2 have true converses. Remember that the converse of a true conditional statement is not necessarily true, so each converse of a theorem must be proved, as in Example 3.

POSTULATE *For Your Notebook*

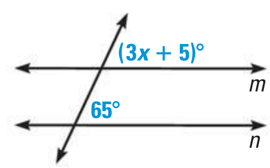
POSTULATE 16 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



EXAMPLE 1 Apply the Corresponding Angles Converse

ALGEBRA Find the value of x that makes $m \parallel n$.



Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

$(3x + 5)° = 65°$ **Use Postulate 16 to write an equation.**

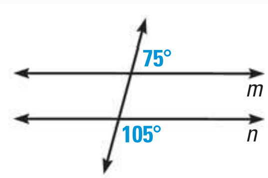
$3x = 60$ **Subtract 5 from each side.**

$x = 20$ **Divide each side by 3.**

▶ The lines m and n are parallel when $x = 20$.

GUIDED PRACTICE for Example 1

1. Is there enough information in the diagram to conclude that $m \parallel n$? *Explain.*
2. *Explain* why Postulate 16 is the converse of Postulate 15.

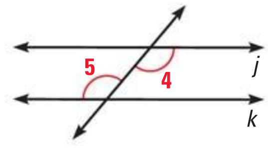


THEOREMS *For Your Notebook*

THEOREM 3.4 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Proof: Example 3, p. 163

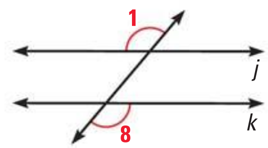


$j \parallel k$

THEOREM 3.5 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Proof: Ex. 36, p. 168

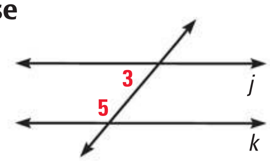


$j \parallel k$

THEOREM 3.6 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

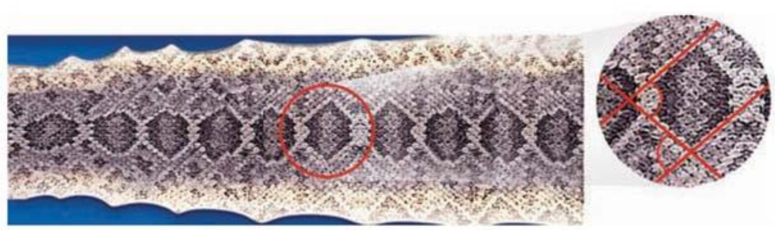
Proof: Ex. 37, p. 168



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

EXAMPLE 2 Solve a real-world problem

SNAKE PATTERNS How can you tell whether the sides of the pattern are parallel in the photo of a diamond-back snake?



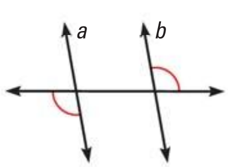
Solution

Because the alternate interior angles are congruent, you know that the sides of the pattern are parallel.

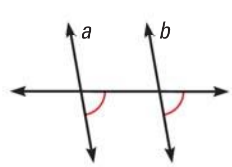
✓ GUIDED PRACTICE for Example 2

Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.

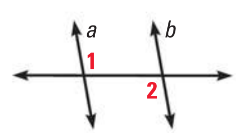
3.



4.



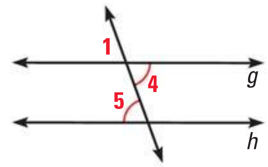
5. $m\angle 1 + m\angle 2 = 180°$



EXAMPLE 3 Prove the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Solution



AVOID ERRORS
Before you write a proof, identify the GIVEN and PROVE statements for the situation described or for any diagram you draw.

GIVEN ▶ $\angle 4 \cong \angle 5$

PROVE ▶ $g \parallel h$

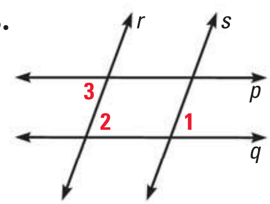
STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Congruence
4. $g \parallel h$	**4.** Corresponding Angles Converse

Animated Geometry at classzone.com

PARAGRAPH PROOFS A proof can also be written in paragraph form, called a **paragraph proof**. The statements and reasons in a paragraph proof are written in sentences, using words to explain the logical flow of the argument.

EXAMPLE 4 Write a paragraph proof

In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

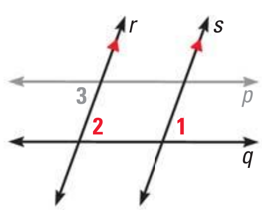


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

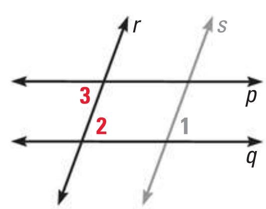
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



$\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$, then $p \parallel q$.

TRANSITIONAL WORDS
In paragraph proofs, **transitional words** such as *so*, *then*, and *therefore* help to make the logic clear.

Plan in Action

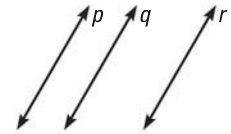
a. It is given that $r \parallel s$, **so** by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. **Then** $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. **Therefore,** by the Alternate Interior Angles Converse, $p \parallel q$.

THEOREM *For Your Notebook*

THEOREM 3.7 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

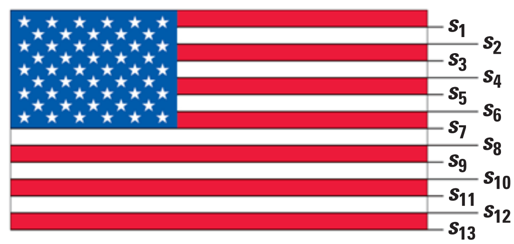


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proofs: Ex. 38, p. 168; Ex. 38, p. 177

EXAMPLE 5 Use the Transitive Property of Parallel Lines

U.S. FLAG The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



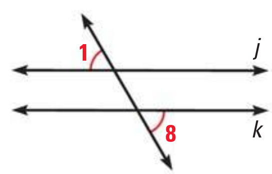
Solution

USE SUBSCRIPTS
When you name several similar items, you can use one variable with subscripts to keep track of the items.

The stripes from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.

✓ GUIDED PRACTICE for Examples 3, 4, and 5

6. If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?



7. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 3.

   It is given that $\angle 4 \cong \angle 5$. By the __?__, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, __?__. So, by the __?__, $g \parallel h$.

8. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. *Explain* why the top step is parallel to the ground.

3.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 29, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 23, 24, 33, and 39

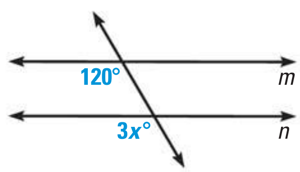
SKILL PRACTICE

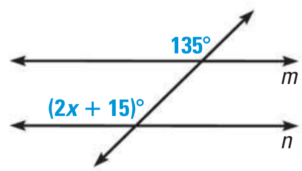
1. **VOCABULARY** Draw a pair of parallel lines with a transversal. Identify all pairs of *alternate exterior angles*.

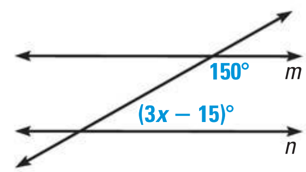
2. ★ **WRITING** Use the theorems from the previous lesson and the converses of those theorems in this lesson. Write three biconditionals about parallel lines and transversals.

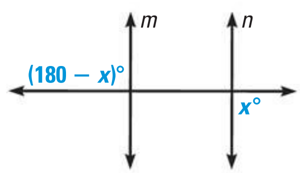
EXAMPLE 1 on p. 161 for Exs. 3–9

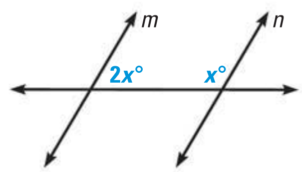
ALGEBRA **Find the value of x that makes $m \parallel n$.**

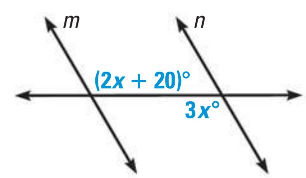
3. 

4. 

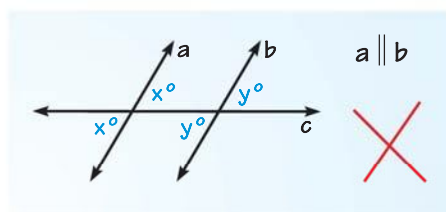
5. 

6. 

7. 

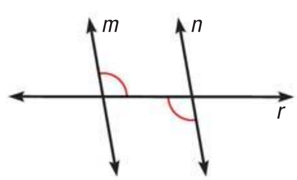
8. 

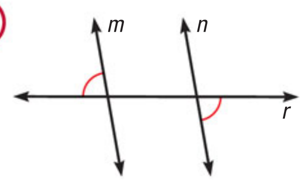
9. **ERROR ANALYSIS** A student concluded that lines a and b are parallel. *Describe* and correct the student's error.

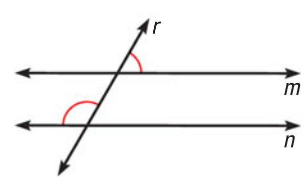


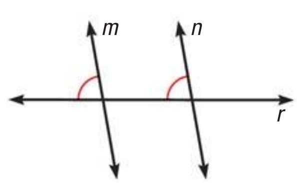
EXAMPLE 2 on p. 162 for Exs. 10–17

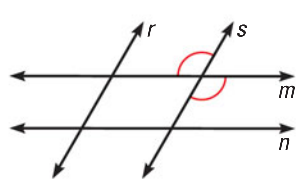
IDENTIFYING PARALLEL LINES **Is there enough information to prove $m \parallel n$? If so, state the postulate or theorem you would use.**

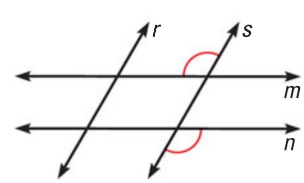
10. 

11. 

12. 

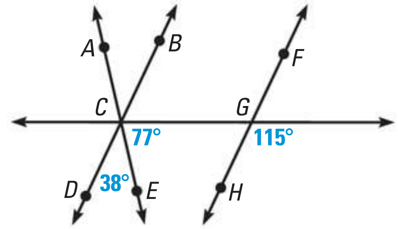
13. 

14. 

15. 

16. ★ **OPEN-ENDED MATH** Use lined paper to draw two parallel lines cut by a transversal. Use a protractor to measure one angle. Find the measures of the other seven angles without using the protractor. Give a theorem or postulate you use to find each angle measure.

17. **MULTI-STEP PROBLEM** Complete the steps below to determine whether $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ are parallel.



a. Find $m\angle DCG$ and $m\angle CGH$.

b. *Describe* the relationship between $\angle DCG$ and $\angle CGH$.

c. Are $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ parallel? *Explain* your reasoning.

EXAMPLE 3
on p. 163
for Ex. 18

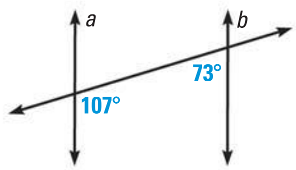
18. **PLANNING A PROOF** Use these steps to plan a proof of the Consecutive Interior Angles Converse, as stated on page 162.

a. Draw a diagram you can use in a proof of the theorem.

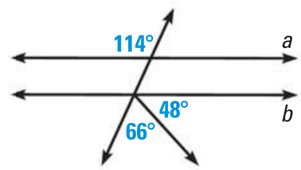
b. Write the GIVEN and PROVE statements.

REASONING **Can you prove that lines *a* and *b* are parallel? If so, *explain* how.**

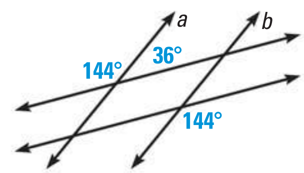
19.



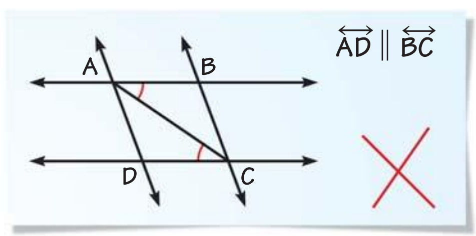
20.



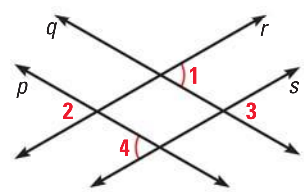
21.



22. **ERROR ANALYSIS** A student decided that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ based on the diagram below. *Describe* and correct the student's error.



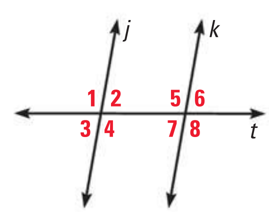
23. ★ **MULTIPLE CHOICE** Use the diagram at the right. You know that $\angle 1 \cong \angle 4$. What can you conclude?



Ⓐ $p \parallel q$ Ⓑ $r \parallel s$

Ⓒ $\angle 2 \cong \angle 3$ Ⓓ None of the above

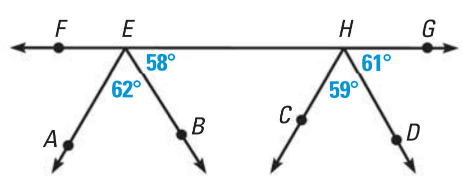
REASONING **Use the diagram at the right for Exercises 24 and 25.**



24. ★ **SHORT RESPONSE** In the diagram, assume $j \parallel k$. How many angle measures must be given in order to find the measure of every angle? *Explain* your reasoning.

25. **PLANNING A PROOF** In the diagram, assume $\angle 1$ and $\angle 7$ are supplementary. Write a plan for a proof showing that lines j and k are parallel.

26. **REASONING** Use the diagram at the right. Which rays are parallel? Which rays are not parallel? *Justify* your conclusions.



27. **VISUAL REASONING** A point R is not in plane ABC.

a. How many lines through R are perpendicular to plane ABC?

b. How many lines through R are parallel to plane ABC?

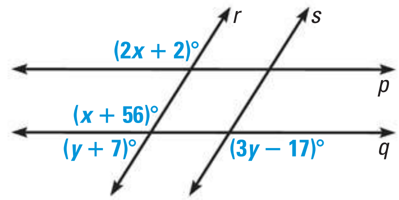
c. How many planes through R are parallel to plane ABC?

28. **CHALLENGE** Use the diagram.

a. Find x so that $p \parallel q$.

b. Find y so that $r \parallel s$.

c. Can r be parallel to s and p be parallel to q at the same time? *Explain.*

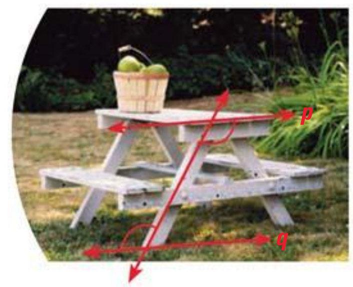


PROBLEM SOLVING

EXAMPLE 2 on p. 162 for Exs. 29–30

29. **PICNIC TABLE** How do you know that the top of the picnic table is parallel to the ground?

@HomeTutor for problem solving help at classzone.com



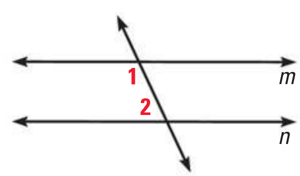
30. **KITEBOARDING** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that n is parallel to m?

@HomeTutor for problem solving help at classzone.com

31. **DEVELOPING PROOF** Copy and complete the proof.

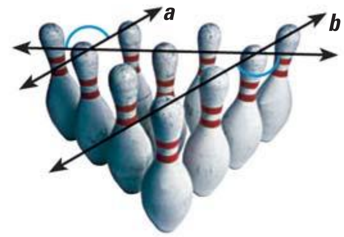
GIVEN ▶ $m\angle 1 = 115°$, $m\angle 2 = 65°$

PROVE ▶ $m \parallel n$



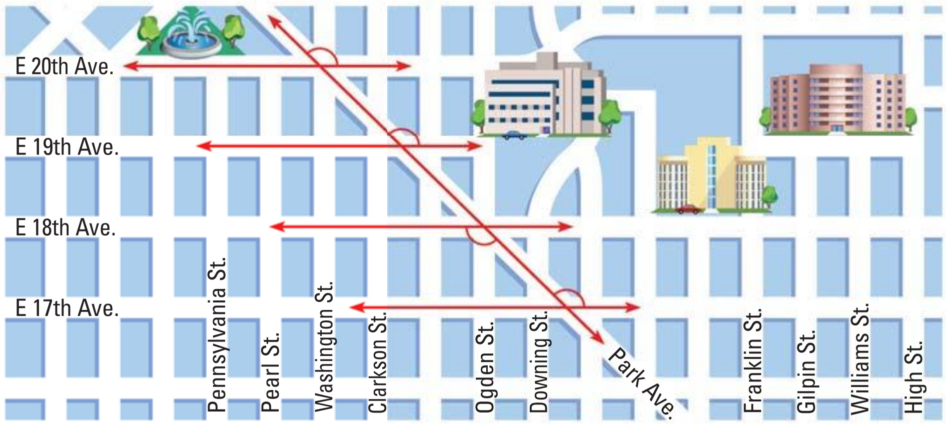
STATEMENTS	REASONS
1. $m\angle 1 = 115°$ and $m\angle 2 = 65°$	1. Given
2. $115° + 65° = 180°$	2. Addition
3. $m\angle 1 + m\angle 2 = 180°$	3. ___?___
4. $\angle 1$ and $\angle 2$ are supplementary.	4. ___?___
5. $m \parallel n$	5. ___?___

32. **BOWLING PINS** How do you know that the bowling pins are set up in parallel lines?



EXAMPLE 5
on p. 164
for Ex. 33

33. ★ **SHORT RESPONSE** The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? *Explain* how you can tell.

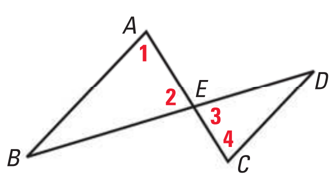


EXAMPLE 3
on p. 163
for Exs. 34–35

PROOF **Use the diagram and the given information to write a two-column or paragraph proof.**

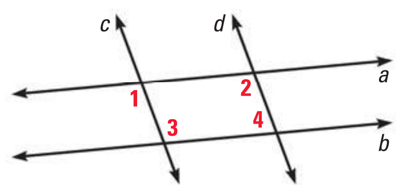
34. **GIVEN** ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\overline{AB} \parallel \overline{CD}$



35. **GIVEN** ▶ $a \parallel b$, $\angle 2 \cong \angle 3$

PROVE ▶ $c \parallel d$

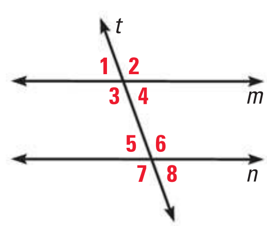


EXAMPLE 4
on p. 163
for Exs. 36–37

PROOF **In Exercises 36 and 37, use the diagram to write a paragraph proof.**

36. **PROVING THEOREM 3.5** Prove the Alternate Exterior Angles Converse.

37. **PROVING THEOREM 3.6** Prove the Consecutive Interior Angles Converse.



38. **MULTI-STEP PROBLEM** Use these steps to prove Theorem 3.7, the Transitive Property of Parallel Lines.

a. Copy the diagram in the Theorem box on page 164. Draw a transversal through all three lines.

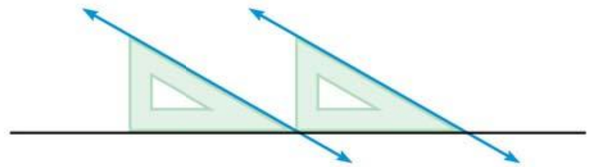
b. Write the GIVEN and PROVE statements.

c. Use the properties of angles formed by parallel lines and transversals to prove the theorem.

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

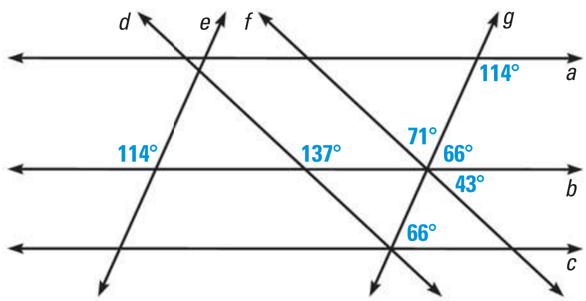
39. ★ **EXTENDED RESPONSE** Architects and engineers make drawings using a plastic triangle with angle measures 30°, 60°, and 90°. The triangle slides along a fixed horizontal edge.



a. *Explain* why the blue lines shown are parallel.

b. *Explain* how the triangle can be used to draw vertical parallel lines.

REASONING **Use the diagram below in Exercises 40–44. How would you show that the given lines are parallel?**



40. a and b

41. b and c

42. d and f

43. e and g

44. a and c

45. **CHALLENGE** Use these steps to investigate the angle bisectors of corresponding angles.

a. Construction Use a compass and straightedge or geometry drawing software to construct line ℓ, point P not on ℓ, and line n through P parallel to ℓ. Construct point Q on ℓ and construct $\overline{PQ}$. Choose a pair of alternate interior angles and construct their angle bisectors.

b. Write a Proof Are the angle bisectors parallel? Make a conjecture. Write a proof of your conjecture.

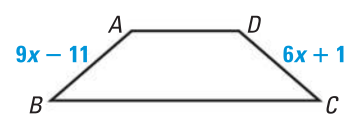
MIXED REVIEW

Solve the equation. *(p. 875)*

46. $\frac{3}{4}x = -1$ **47.** $\frac{-2}{3}x = -1$ **48.** $\frac{1}{5}x = -1$ **49.** $-6x = -1$

50. You can choose one of eight sandwich fillings and one of four kinds of bread. How many different sandwiches are possible? *(p. 891)*

51. Find the value of x if $\overline{AB} \cong \overline{AD}$ and $\overline{CD} \cong \overline{AD}$. *Explain* your steps. *(p. 112)*



PREVIEW
Prepare for Lesson 3.4 in Exs. 52–54.

Simplify the expression.

52. $\frac{-7-2}{8-(-4)}$ *(p. 870)* **53.** $\frac{0-(-3)}{1-6}$ *(p. 870)* **54.** $\frac{3x-x}{-4x+2x}$ *(p. 139)*